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# MATHEMATICAL MODELING OF THE ABILITY TO DETECT AND MEASURE SMALL LEAKS IN PIPELINE SYSTEMS

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Minor leaks occur in the case of pipeline operation – axis curvature, the effect of force factors of various nature that affect the change in the stress-strain state up to the emergence of zones potentially dangerous in terms of possible violation of material integrity, the presence of surface defects that can cause liquid leakage. In this case, the loss of the transportation product is relatively small – up to 0.1-0.5%. Defects of this kind are determined by experimental methods, but the use of mathematical modeling methods for phenomena of this nature is becoming increasingly important, in particular, due to the complexity of hardware implementation due to the difficulty of accessing surfaces for the implementation of contact research methods.

**Problem statement.** When studying the technical condition of complex mechanical systems that have been in operation for a long time, in particular in the tasks of their technical diagnostics, there are often cases when the occurrence of emergency situations is due to the presence of small disturbances acting on the system. In mathematical modeling of such phenomena, as a rule, there are problems in which either systems of equations or boundary and initial conditions are subjected to certain perturbations, the characteristics of which are small compared to the characteristic values of the quantities under study, since large perturbations of the initial conditions usually necessitate the choice of other models to describe the phenomena - a commentary on this can be made by models for estimating flow parameters with small leaks and models of flow at pipeline rupture [1].

It should be noted that the study of hydrodynamic stability in the zone of small leaks is important from two aspects affecting energy efficiency: firstly, the amount of transported hydrocarbon loss is estimated, and secondly, the flow structure in the presence of small leaks of varying intensity is studied in terms of the emergence of turbulent flow zones, which can actually reduce the useful diameter of the pipeline.

It is proposed to consider approaches to the use of process stability parameters to solve a specific technical problem of assessing the technical condition of pipeline systems, taking into account the fact that numerical methods are used in the implementation of these models. This raises questions about the possibility of numerically identifying the critical parameters at which a certain physical process loses stability if the characteristics of this process are not known and are actually a solution to a certain boundary value problem.

Thus, the aim of the study is a mathematical modeling of process fluid flow in a pipeline in the presence of leaks through the surface at different configurations, for which the system of Navier-Stokes equations is numerically integrated, the stability parameters of numerical schemes are investigated, informative parameters are selected to determine the leakage zones of influence, and the limits of the model's use are set before the flow transitions to the turbulent regime.

**Mathematical modeling.** In general, the flow of products in pipelines can be described using the Navier-Stokes system of equations written in a cylindrical coordinate system [2-3]. However, the peculiarities of pipeline systems in terms of their geometry, as well as the symmetry of the flow, the local nature of the zone of small leaks, allow us to reduce the dimensionality of the problem. Thus, it can be assumed that we are considering a two-dimensional flow of a viscous fluid in a channel with a wall in which there is a leakage of fluid through the surface, assuming that the flow is considered stationary. This is true, in particular, for quasi-stationary processes, when it is assumed that the characteristics of the modeled flow change little with time (Fig. 1).

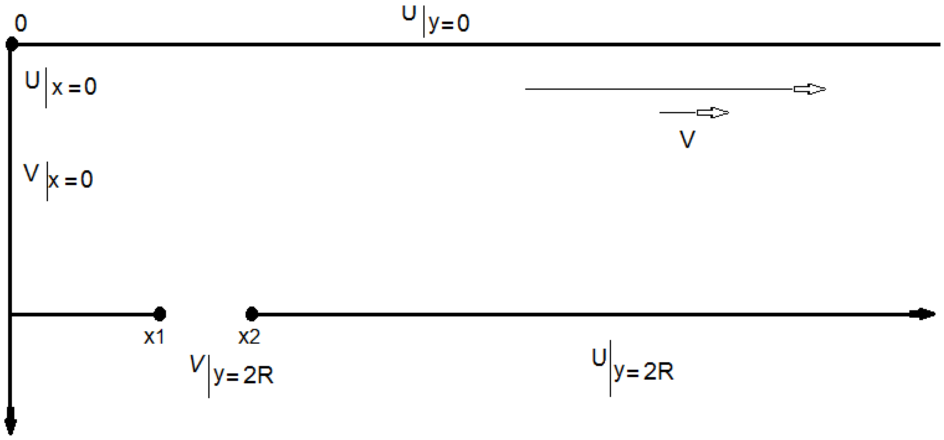


Fig.1 Flow diagram in a two-dimensional channel with leaks

In this case, the system of Navier-Stokes equations is written in the two-dimensional domain as follows:

$$\begin{cases} U \cdot \frac{\partial U}{\partial x} + V \cdot \frac{\partial U}{\partial y} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \nu \cdot (\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}) \\ U \cdot \frac{\partial V}{\partial x} + V \cdot \frac{\partial V}{\partial y} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial y} + \nu \cdot (\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}) \\ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \end{cases} \quad (1)$$

where $U$ and $V$ – components of the velocity vector in a rectangular Cartesian coordinate system, $\rho$ – density of the transported products, $\nu$ – coefficient of kinematic viscosity, $p$ – fluid pressure.

The boundary conditions are given in the form:

$$\begin{cases} U\Big|x = 0 = -\frac{k \cdot y^2}{4 \cdot \mu} + \frac{k \cdot R_y}{2 \cdot \mu} \\ U|y = 0 = U|y = 2 \cdot R = 0 \\ V|x = 0 = V|y = 0 = 0 \\ V\Big|y = 2 \cdot R = \begin{cases} 0 x < x_1, x < x_2 \\ V_{laek} x \in [x_1; x_2] \end{cases} \end{cases} \quad (2)$$

where $[x_1; x_2]$ – leakage zone, $\mu$ – dynamic viscosity of the transported products, $R$ – channel radius. For the velocity component $U|x = 0$ is believed to be calculated as in the well-known Poiseuille model [2], which describes the stationary flow of a viscous fluid in a pipe of circular cross-section. $V_{leak}$ – fluid leakage rate through the area.

Boundary conditions (2) may be different depending on how the fluid leakage zones are located - if they are located on different channel boundaries, then for the velocity component *V*, the velocity values at certain segments will be non-zero both at *y=0* and at *y=2R*. The methodology for solving this problem is known [4], and its peculiarity is the presence of discontinuous boundary conditions (2) and the absence of correct boundary conditions for pressure.

Differentiating the first of the equations of system (1) by the variable $x$, and the second - by the variable $y$, and taking into account the third equation of system (1), the Poisson's equation for determining the pressure is obtained:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -2 \cdot \rho (\frac{\partial V}{\partial x} \cdot \frac{\partial U}{\partial y} - \frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y}) \quad (3)$$

Further solution scheme is as follows:

1) some initial pressure approximation $p_0(x, y)$ is set;

2) according to this distribution $p_0(x, y)$ system (1) with boundary conditions (2) shall be solved;

3) after finding the velocity components $U$ i $V$ the right-hand sides of equation (3) shall be calculated;

4) equation (3) shall be solved with boundary conditions:

$$p|\partial G = p_0(x, y) \quad (4)$$

5) after obtaining a new pressure distribution, the specified algorithm returns to step 1.

This procedure is repeated until the iterative process reaches convergence. System (1) with boundary conditions (2) is solved using absolutely convergent implicit schemes of the method of alternating directions, and equation (3) is solved by the method of successive upper relaxation. The convergence and stability of this iterative method were proved in [2].

The initial approximation of the pressure distribution was chosen under the assumption that there is a linear pressure drop along the length of the channel used to model the leaking pipe:

$$p = p_0 - k \cdot x \quad (5)$$

Using dependence (5) to calculate the velocity field allows us to establish the correlation between the leakage rate and the change in the flow configuration.

Flow modeling in a pipeline with defects that cause fluid outflow is performed for the following flow parameters, pipe geometry, properties of liquids and gases, and linear pressure drop along the length of the pipe:

1) the average velocity of the liquid in the pipeline is 2-8 m/s;
2) characteristic velocity of small leakage - up to 50 cm/s;
3) dynamic liquid viscosity - 0.001 kg/m/s;
4) kinematic viscosity - 0.000001 sq. m./sec;
5) characteristic of the pressure drop K= 0.064 - 0.096;
6) step in the longitudinal coordinate - 0.08 m;
7) step in the longitudinal coordinate - 0.025 m, which corresponds to a pipeline with a diameter of 1.25 m with 50 control points in the transverse coordinate;
8) the number of steps in the longitudinal coordinate is 90000, which allows to calculate the velocity field for a 7.2 km long pipe with a step of 8 cm.

**Results.** Based on the calculations, the following conclusions can be drawn: the leakage rate significantly affects the flow configuration.

Analyzing the behavior of the longitudinal velocity component in the near-wall zone, we can note a pattern that depends on the leakage rate: the higher the leakage rate, the faster the monotonicity of the velocity field on the leakage side is broken (Fig. 2). In addition, the following regularity was found: a violation of monotonicity, which can be defined as the difference in velocity at the two points of the grid closest to the wall:

$$\Delta V_m = V(N) - V(N-1) \quad (6)$$

where $N+1$ - the number of points of the computational grid along the transverse coordinate, follows the following pattern: initially, the first monotonicity violation occurs, then monotonicity is restored, and its subsequent loss leads to a loss of stability of the computational process, as shown schematically in Fig. 2.

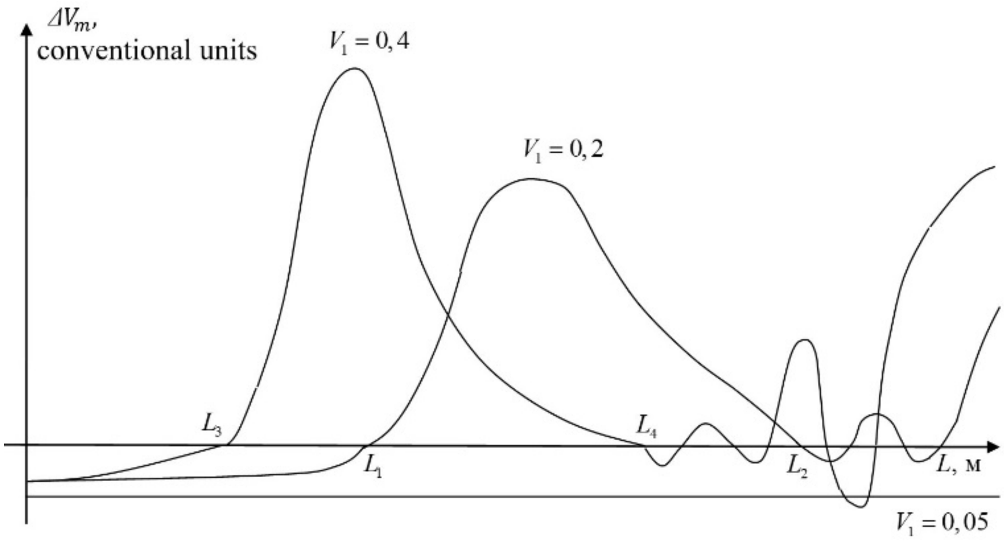


Fig.2 Dependence between $\Delta V_m$ and distance from the defect at different model values of the leakage rate, given in conventional units

Points $L_3$ and $L_1$ can serve as the flow response to a small disturbance, they correspond to the minimum distance at which the disturbance is already felt, and points $L_4$ and $L_2$ are the points of loss of stability of the difference circuit. In this case, the points $L_3$ and $L_1$ can serve as a diagnostic sign, but $L_4$ i $L_2$ – cannot. In this case, the content of the processes occurring after these points may be as follows: either the stability of the computational procedure is lost, or the physical picture of the flow changes - from laminar to turbulent, and other models must be used to further describe the flow. From a technical point of view, this behavior can be explained by the fact that when a fluid is slowed down along the length of a pipe, it must be pumped to ensure a certain pressure, flow rate, and, accordingly, the specified delivery volume. This leads to a decrease in the energy efficiency of the pipeline system. It is not known which content should be prioritized in each case, and further research into these flows is needed. An important result shown in Figs. 2 and 3 is that at certain values of the leakage velocity - in Fig. 2, this value is V = 0.05 - there is no loss of velocity monotonicity at all, i.e. the flow remains stable to such a velocity perturbation.

**Conclusions.** Based on the results of numerical modeling of fluid flow through a channel with leakage through the surface, a method for estimating the coordinate of the leakage point and its dependence on the leakage rate is established. It has been confirmed that the problems of technical diagnostics of systems for various purposes are, from a mathematical point of view, problems of studying the stability of the corresponding processes and numerical schemes for implementing models of such processes.

The presence of small leaks on the energy efficiency of the system causes the following negative aspects:

- loss of transported products;

- loss of flow stability in the pipeline, emergence of turbulent (unstable) flow zones, which actually reduces the actual useful diameter of the pipe and, accordingly, its actual throughput.

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# ВДОСКОНАЛЕННЯ КОНСТРУКЦІЇ ЧЕРВ'ЯЧНОГО ЕКСТРУДЕРА ДЛЯ ПІДВИЩЕННЯ ЕФЕКТИВНОСТІ ВИРОБНИЦТВА ПОЛІМЕРНИХ ЛИСТІВ

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Екструзійні технології відіграють центральну роль у сучасному промисловому виробництві, особливо у сфері переробки полімерних матеріалів. Їх основна функція полягає в точному та контрольованому перетворенні сировини у кінцевий продукт, що досягається шляхом застосування контрольованого тепла, тиску та механічної енергії. Черв'ячні екструдери, будучи ключовим елементом цих технологій, є незамінними у виробництві широкого спектру продукції, включаючи полімерні листи. Ці листи знаходять своє застосування в будівництві, пакуванні, сільському господарстві, рекламі та багатьох інших галузях, що підкреслює їхню універсальність та важливість для сучасної економіки.